JUST INTONATION

A BASIS FOR ENHANCING CHORAL INTONATION

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The human voice is the most beautiful instrument of all, but it is the most difficult to play.

—Richard Strauss, n.d.

Poor, unreliable, inconsistent intonation is a significant problem for choristers and their directors. Choirs sing out of tune for various reasons, but one that receives little attention is the negative effects of musical temperaments, such as when the piano is used to verify intonation during rehearsals. This practice typically results in the temperament of the piano conflicting with the natural tuning of the voice.

A better option is to tune unaccompanied music, particularly early music, according to the principles of just intonation—an approach in which sung pitches are matched to overtones of the harmonic series. The resultant sound is perceived as more resonant and in tune, as there are minimal beats[1] between the sung notes, harmonics, and resultant tones.[2] J. M. Barbour, in writing about just intonation, states that "it properly means singing or playing in tune,"[3] and J. Montegu regards it as the pinnacle of perfect tuning.[4] N. L. Norden states:

> A cappella music being based upon the natural laws of the universe [just intonation], brings us nearer to the Center [sic], or God. It is not man-altered: it is based upon eternal, unchanging laws. For this reason it has a beauty and a charm that are unchallengeable.[5]

Although this statement appears to have theological and subjective connotations, it has relevance in the secular context of choral intonation. Per-Gunnar Alldahl implies that just intonation seems appropriate for unaccompanied music written in the twelfth to seventeenth centuries when he states that "perfect, non-beating, triads may be an ideal to strive for in a cappella singing—with 'free' intonation—especially in older music from the Renaissance and Baroque."[6] Although there is evidence to suggest that just intonation could be used to tune unaccompanied tonal music from other periods, that subject is beyond the scope of this introductory article on just intonation in the choral context. It is important to note that there may be instances when the use of just intonation is unsuitable—for example, in some folk music. It may also be less effective for tonally ambiguous, atonal and serial music, as just intonation establishes a hierarchy among notes, which is in conflict with the intent of music where no pitch has superiority over others.

Just intonation can offer a solution to solving tuning problems, but at present its use in choral rehearsals is extremely rare, likely because there is some debate among just intonation experts as to its application, the mathematics are admittedly complex, there is currently no user-friendly way to teach it to choirs, and because we are immersed in temperaments (particularly Equal Temperament) and have been so for approximately two hundred years. This introductory article[7] shows how just intonation can be used as a tool for tuning unaccompanied tonal music and outlines one approach in teaching it to choirs. Jacob Handl's *Resonet in Laudibus* is used as a case study (see page 40-41).

Why Just Intonation?

The just intonation of a note is attained by either matching its intonation to a corresponding audible harmonic of its root, or tuning it in such a way that it forms a natural ratio with its root. While there are an infinite number of harmonics, the intonations of twelve of the first sixteen are the most useful in a choral context, as four produce unsatisfactory results (these are crossed [x] in Figure 1).

Thus, in just intonation, the C-Major chord (Figure 2) is tuned using harmonics of the note c3: the bass is the fundamental and first harmonic (0 cents), the soprano c5 is tuned to the fourth harmonic (also 0 cents), the tenor g3 is tuned to the third harmonic (701.955001 cents), and the alto e4 is tuned to the fifth harmonic (386.313714 cents).

By tuning a major chord in this way, beats are minimized between the sung notes, resultant tones, and harmonics ("neighbouring harmonics"[8]), as there is mostly only one intonation of each pitch class. The resultant sound is perceived as resonant (often louder) and perfectly in tune. K. D. Skelton states that "when harmonics between pitches coincide, the result is pleasing to singers and audience alike. It feels right."[9] This is not the case with temperaments, and many researchers argue that (a) all notes are out of tune except the octave,[10] (b) beats occur,[11] and (c) there is less consonance.[12] For example, when the same C-Major chord is tuned in Equal Temperament, there is a greater number of beats since there are two intonations of the notes: "c," "g," and "e"; and three intonations of "d," "f-sharp," and "b."

The just intonation of each scale degree in comparison to Equal Temperament is shown in Table 1. One of the most distinguishing features of just intonation is that alternative intonations of pitch classes are required, such as the major second and minor seventh scale degrees (highlighted in Table 1). For example, in the key of F Major, the note "g" (major second scale degree) could be 182.403712 or 203.910002 cents depending on its harmonic context, and the note "e-flat"

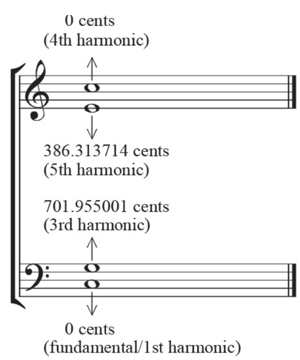

Figure 2. C-Major chord

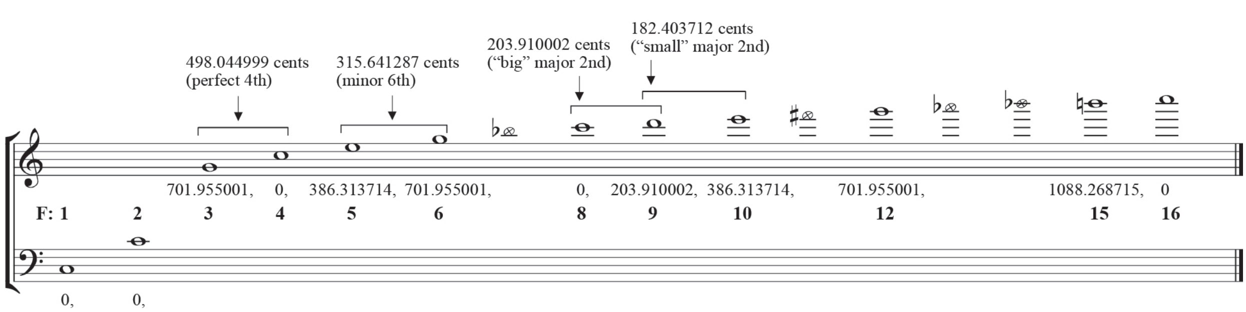

Figure 1. The intonations (as cents) of twelve of the first sixteen harmonics of c3.

Table 1. Comparison of Commonly Used Just Ratios and Intonations with Equal Temperament

Scale Degree	Ratio	Just Intonation (in cents)	Equal Temperament (in cents)	Difference (in cents)
ORIGIN: Perfect Unison (PU)	1:1	0	0	0
Diminished Second (d2)	128:125	41.058858	0	+41.058858
Augmented Unison (AU)	25:24	70.672427	100	-29.327573
Minor Second (m2)	16:15	111.731285	100	+11.731285
Major Second (M2) – small	10:9	182.403712	200	-17.596288
Major Second (M2) – big	9:8	203.910002	200	+3.910002
Diminished Third (d3)	144:125	244.968860	200	+44.968860
Augmented Second (A2)	75:64	274.582429	300	-25.417571
Minor Third (m3)	6:5	315.641287	300	+15.641287
Major Third (M3)	5:4	386.313714	400	-13.686286
Diminished Fourth (d4)	32:25	427.372572	400	+27.372572
Augmented Third (A3)	125:96	456.986141	500	-43.013859
Perfect Fourth (P4)	4:3	498.044999	500	-1.955001
Augmented Fourth (A4)	45:32	590.223716	600	-9.776284
Diminished Fifth (d5)	64:45	609.776284	600	+9.776284
Perfect Fifth (P5)	3:2	701.955001	700	+1.955001
Diminished Sixth (d6)	192:125	743.013859	700	+43.013859
Augmented Fifth (A5)	25:16	772.627428	800	-27.372572
Minor Sixth (m6)	8:5	813.686286	800	+13.686286
Major Sixth (M6)	5:3	884.358713	900	-15.641287
Diminished Seventh (d7)	128:75	925.417571	900	+25.417571
Augmented Sixth (A6)	125:72	955.031140	1000	-44.968860
Minor Seventh (m7) – small	16:9	996.089998	1000	-3.910002
Minor Seventh (m7) – big	9:5	1017.596288	1000	+17.596288
Major Seventh (M7)	15:8	1088.268715	1100	-11.731285
Diminished Octave (d8)	48:25	1129.327573	1100	+29.327573
Augmented Seventh (A7)	125:64	1158.941142	1200	-41.058858
ORIGIN: Perfect Octave (P8)	2:1	1200 (0)	1200 (0)	0
Cent values calculated using http://www.sengpielaudio.com/calculator-centsratio.htm				

(minor seventh) could be 996.089998 or 1017.596288 cents. M. Sabat mentions alternative intonations of pitch classes,[13] but Norden implies that these are seldom utilized any more.[14] This may be due to the complexity associated with determining which intonations to use.[15] Choristers can achieve different intonations of pitch classes because their voices are capable of minute changes in pitch.

The intonation of a pitch class is determined by its harmonic context. In Figure 3, the alto g4 is 182.403712 cents when it is functioning as the third of the E-flat-Major chord, but 203.910002 cents when it is functioning as the perfect fifth of the C-Minor chord. Similarly, the tenor e-flat4 is 996.089998 cents when it is functioning as the root of the E-flat-Major chord, but 1017.596288 cents when it is functioning as the minor third of the C-Minor chord. Determining which version of the major second and minor seventh to use will be discussed later in the article.

Background

Conductors spend considerable time correcting tuning problems in many different ways. Instructions such as "widen ascending intervals," "narrow descending intervals," and "make the intonation of repeated notes the same" are often heard during rehearsals. These strategies, however, are not systematic or applicable to multiple contexts, nor do they have a long-term effect on improving intonation; in fact, some are unintentionally compromising it!

If the instructions above are applied to Figure 3, perfect intonation would be unattainable, as the ascending interval from g4 to a4 (soprano, bar 6) should be narrower than in Equal Temperament (182.403712 cents, rather than 200), the descending interval from c5 to b-flat4 (soprano, between bars 5 and 6) should be wider (203.910002 cents, rather than 200 cents), and the tuning of the alto's fourth g4 (bar 5) should be different from the first three (203.910002 cents, rather than 182.403712). Note that cent values are used to represent the intonation of notes, as these are generally easier for conductors and choristers to comprehend than Hertz or ratios. There are 1200 cents in an octave and 100 in an equally tempered half tone. For clarity, cent values greater than 1200 are reduced so that they are within an octave range; that is, octave displacements are omitted. For example, a major ninth is written as 203.910002 cents rather than 1403.910002 because its intonation is equivalent to a major second.

It is desirable to tune non-harmony notes purely to obtain true just intonation; if not, the result is something other than just intonation. In fact, for all harmony and non-harmony notes to be perfectly in tune and just intonation to be fully implemented, they need to be pitched according to their vertical harmonic context. As a consequence, there may be more than one intonation of some notes (pitch classes) in a piece of music, such as the major second and minor seventh scale degrees. Further, in just intonation, the tuning of identical pitches may need to change so they remain in tune within different harmonic contexts. Without changes to some of the notes in Figure 3, just intonation would not be completely possible. For example, the second tenor e-flat4 in bar 5 is sharper (1017.596288 cents) than the first (996.089998 cents) when it changes function from the root of an E-flat-Major chord to the minor third of a C-Minor chord.

In defence of conductors, however, the literature on choral pedagogy is full of generic solutions such as those previously mentioned, and as Marvin states, "of all the challenges of the choral art, achieving good intonation

Figure 3. Jacob Handl, *Resonet in Laudibus,* Extract, mm. 5–6.

is probably the most elusive."[16] Further, determining how a note should be tuned may be challenging for conductors if they have a limited understanding of tuning systems.[17]

Pedagogical Approach

While just intonation has been shown to be a useful tool in the enhancement of choral tuning, there is not a simple approach for teaching it to choristers. Following is one systematic, comprehensible approach to teach just intonation tuning techniques to choirs, which has been successfully used with two choirs in New Zealand.

Just Intonation Tuning Symbols

There is an immediate issue with attaining just intonation: "The guide for the just interval is the absence of any beats…left alone, [the chorister] cannot find them except by chance."[18] This "chance" factor is mentioned in the 2013 book *Choral Pedagogy*[19] and by J. Grant, who states that "correctly tuned intervals are not achieved by chance. The singers must have a way of thinking about this process; a system in which to work."[20]

The following series of symbols has been devised for this purpose. Just intonation tuning symbols[21] (Table 2) show choristers the degree to which notes need to be brightened (sharpened) or relaxed (flattened).[22] Like road maps and compasses, the symbols only point singers in the right direction; the singers then have to develop and use their aural (listening for an absence of

Table 2. Just Intonation Tuning Symbols

Symbols	Descriptors and Difference from Equal Temperament
▲	Extremely bright: 32.259435 cents (1½ syntonic commas[23]) to 43.01258 cents (2 syntonic commas) sharper, or more.
△	Very bright: 21.50629 cents (1 syntonic comma) and less than 32.259435 cents (1½ syntonic commas) sharper.
^	Bright: 10.753145 cents (½ syntonic comma) and less than 21.50629 cents (1 syntonic comma) sharper.
↗	Slightly bright: greater than 0 cents and less than 10.753145 cents (½ syntonic comma) sharper.
○	Origin (centred): 0 cents and equivalent to equal temperament or the external source, such as the piano.
↘	Slightly relaxed: less than 0 cents and greater than -10.753145 cents (½ syntonic comma) flatter.
v	Relaxed: -10.753145 cents (½ syntonic comma) and greater than -21.50629 cents (1 syntonic comma) flatter.
▽	Very relaxed: -21.50629 cents (1 syntonic comma) and greater than -32.259435 cents (1½ syntonic commas) flatter.
▼	Extremely relaxed: -32.259435 cents (1½ syntonic commas) to -43.01258 cents (2 syntonic commas) flatter, or less.
Cent values calculated using http://www.sengpielaudio.com/calculator-centsratio.htm	

beats in the sound) and sensory (feeling the vibrations of an in-tune sound) skills to verify when the just intonation is harmonically achieved. When the sound of just intonation is securely embedded in choristers' ears, there should be no need for symbols.[24]

In open scores, the symbols are placed above each vocal line; in short SATB scores, they are placed above the soprano and tenor notes, and beneath the alto and bass. The symbols also function in a similar way to accidentals—that is, they last for the duration of a bar, over tied notes, and are rewritten for different octaves. Choristers are not required to learn the mathematics of just intonation, such as the cent values of notes. All that is required of singers is an ability to use the symbols to guide the ear to the just intonation of notes.

When just intonation tuning symbols were taught to two choirs in New Zealand, choristers in both choirs quickly became proficient at reading the symbols, and their fluency consistently improved. This view was corroborated by the artistic staff involved with one of the choirs, and 93.9% of participants reported positively about their ability to read the symbols (Table 3). Many commented that this was because the symbols were frequently referred to in rehearsals and were logical and clear. Choristers became quicker at tuning notes when they were reading the symbols, which also increased their motivation and determination to succeed. The cent gradations between the symbols were modified during this, and subsequent, research.

Score Preparation Process for Conductors

Thorough score preparation is essential in being able to attain accurate just intonation results with choristers in rehearsals. This process begins with completing a harmonic and tonal analysis (shown as a root line on a score—see *Resonet in Laudibus* on pages 40-41) and then using Tables 1 and 2 to calculate just intonation for, and assign symbols to, each note.[25] While the following score preparation process may appear daunting and time-consuming for conductors, the results in rehearsals and performances are worthwhile.

Calculating Just Intonation and Assigning Symbols

The origin (tonal center) of *Resonet in Laudibus* is "f." In just intonation, the tuning of the origin is always 0 cents, and Table 1 can be used to determine the intonation of the roots of chords according to their intervallic relationship with the origin. For example, the intonation of the

Table 3.

How would you rate your ability to read just intonation tuning symbols?					
Categories	**Choir 1**	**Choir 2**	**Staff**	**Total**	**Grouped Responses**
1: Excellent	13	2	1	16	**93.9%**
2: Very good	24	9	0	33	
3: Good	17	10	1	28	
4: Fair	1	2	0	3	**6.1%**
5: Poor	1	1	0	2	
6: No answer	0	0	0	0	**0%**
Number of Participants	56	24	2	82	**100%**

root of a "c" chord is 701.955001 cents (1.955001 cents sharper than its equally tempered equivalent) because its relationship with the "f" origin is a perfect fifth. Further, the just intonation of notes is calculated harmonically from the root to (a) achieve a hierarchical relationship between pitches, (b) maintain consistency in the score preparation process, and (c) ensure that the distinct colors of different chords are attained. Calculating the just intonation of notes melodically is not recommended. Melodic intonation, which occurs from one note to the next, happens as a result of tuning harmonically. Note that inversions do not change the tuning; the notes of chord Ia have the same intonation as Ib and Ic.[26]

Roots

A root's (or implied root's) intonation, and subsequent symbol, is determined by its intervallic relationship with the origin. Roots are bracketed when they are not in the lowest vocal part, because choristers need to be aware of the tuning reference note—for example, the tenor "d4" in bar 7. Both J. M. Jordan & M. Mehaffey[27] and T. Wine[28] stress the importance of choristers being aware of the root. Cent values of roots are calculated using the following formula:

ORIGIN + SCALE DEGREE (from Table 1) =
ROOT ➔ SYMBOL (from Table 2)

Example 1

In bar 1, the "f" root is the origin, which is 0 cents (Table 1). It is assigned an origin (O) symbol (Table 2).

Example 2

In bar 1, the "d" root is functioning as the major sixth scale degree of the "f" origin (Table 1): 0 ("f" origin) + 884.358713 (M6 scale degree) = 884.358713 cents ("d" root). It is assigned a relaxed (∨) symbol (Table 2) because its intonation is 15.641287 cents flatter (less) than its equally tempered equivalent—884.358713 cents instead of 900.

Other Notes of Chords

The intonation of all other harmony and non-harmony notes is calculated from the roots of chords. Cent values of notes are calculated using the following formula:

ROOT + SCALE DEGREE (from Table 1) =
NOTE ➔ SYMBOL (from Table 2)

Example 3

The notes of the F-Major chord in bar 1 are calculated as follows:

- a: 0 ("f" root) + 386.313714 (M3 scale degree) = 386.313714 cents ➔ ↘ symbol
- c: 0 ("f" root) + 701.955001 (P5) = 701.955001 cents ➔ ↗

Example 4

The notes of the D-Minor chord in bar 1 are calculated as follows:

- f: 884.358713 ("d" root) + 315.641287 (m3) = 1200. 1200 − 1200 = 0 cents ➔ O. *1200 has been subtracted from 1200, so the cent value is within an octave range, that is, between 0 and 1199.*
- a: 884.358713 ("d" root) + 701.955001 (P5) = 386.313714 cents ➔ ∨

The Major Second Scale Degree

As mentioned earlier, in just intonation there are two versions of the major second scale degree of the origin (Table 1.) Which version to use is determined by the harmonic context and the maintenance of the just relationships between the notes and with the origin.

Example 5

The notes of the G-Minor chord in bar 5 are calculated as follows:

- g: 0 ("f" origin) + 182.403712 (a small M2 scale degree) = 182.403712 cents ➔ ∨
- b-flat: 182.403712 ("g" root) + 315.641287 (m3) = 498.044999 cents ➔ ↘

- d: 182.403712 (“g” root) + 701.955001 (P5) = 884.358713 cents ➔ ∨

A “g3” root tuned to 203.910002 (a big M2 scale degree) would compromise the just relationships between the notes and with the origin. The just relationship between the “g” and the “b-flat” would be 294.134997 cents rather than a just minor third (315.641287); similarly, the tuning between the “g” and the “d” would be 680.448711 cents rather than a just perfect fifth (701.955001).

Example 6

The notes of the C-Major chord in bar 7 are calculated in the following example. Here, the major second scale degree of the origin is functioning as the perfect fifth of the chord and needs to be big—203.910002 cents:

- c: 0 (“f” origin) + 701.955001 (P5 scale degree) = 701.955001 cents ➔ ↗

- e: 701.955001 (“c” root) + 386.313714 (M3) = 1088.268715 cents ➔ ∨

- g: 701.955001 (“c” root) + 701.955001 (P5) = 1403.910002. 1403.910002 – 1200 = 203.910002 cents ➔ ↗. *1200 has been subtracted from 1403.910002 so the cent value is within an octave range (between 0 and 1199).*

A “g” tuned to 182.403712 (a small M2 scale degree) would compromise its just relationship with the “c”; the distance between them would be 680.448711 cents rather than a just perfect fifth (701.955001).

The Minor Seventh Scale Degree

There are also two versions of the minor seventh scale degree—the major second’s complementary interval.

Example 7

The notes of the E-flat-Major chord in bar 5 are calculated as follows:

- e-flat: 0 (“f” origin) + 996.089998 (a small m7 scale degree) = 996.089998 cents ➔ ↘

- g: 996.089998 (“e-flat” root) + 386.313714 (M3) = 1382.403712 ➔ 182.403712 cents ➔ ∨

- b-flat: 996.089998 (“e-flat” root) + 701.955001 (P5) = 1698.044999 ➔ 498.044999 ➔ ∨

An “e-flat” root tuned to 1017.596288 (a big m7 scale degree) would compromise the just relationships between the notes and with the origin. The just relationship between the “e-flat” and the “g” would be 364.807424 cents rather than a just major third (386.313714). Similarly, the tuning between the “e-flat” and the “b-flat” would be 680.448711 cents rather than a just perfect fifth (701.955001).

Example 8

The notes of the C-Minor chord in bar 5 are calculated below. In this example, the minor seventh scale degree of the origin is functioning as the minor third of the chord and needs to be big—1017.596288 cents:

- c: 0 (“f’ origin) + 701.955001 (P5 scale degree) = 701.955001 cents ➔ ↗

- e-flat: 701.955001 (“c” root) + 315.641287 (m3) = 1017.596288 cents ➔ ∧

- g: 701.955001 (“c” root) + 701.955001 (P5) = 1403.910002 ➔ 203.910002 cents ➔ ↗

An “e-flat” tuned to 996.089998 (a small m7 scale degree) would compromise its just relationship with the “c”; the distance between them would be 294.134997 cents rather than a just minor third (315.641287).

Non-harmony Notes

To realize just intonation fully and maximize the resonance of chords, the tuning of non-harmony notes needs careful consideration.

Example 9

The “g” passing note in bar 9 (tenor) needs to be 203.910002 cents so that this tuning matches the harmonics of the other sung notes, specifically, the ninth harmonic of the "f" and the third, sixth, and twelfth harmonics of the "c." If "g" is 182.403712 cents, beats would occur with these harmonics and compromise the resonance of the chord.

Example 10

Similarly, the "c3" passing note (bass) in bar 9 needs to be 701.955001 rather than 680.448711, so that it matches the third, sixth, and twelfth harmonics of the "f."

Secondary Dominant Chords

It is preferable to tune the root of secondary dominant chords to the root of the chord in which they tonicize so that a just perfect fifth relationship is achieved between them (see Examples 11 and 12). Alternatively, conductors may choose to tune secondary dominant chords to the origin instead.

Example 11

In bar 11, the intonation of the root of chord V/V (G-Major) is calculated from the root of chord V (C-Major): 701.955001 (root of V) + 701.955001 (P5) = 1403.910002 → 203.910002 cents (root of V/V). The other notes are calculated as follows:

- b: 203.910002 (root of V/V) + 386.313714 (M3) = 590.223716 cents → ↘

- d: 203.910002 (root of V/V) + 701.955001 (P5) = 905.865003 cents → ↗

A "g" root tuned to 182.403712 (a small M2 scale degree) would compromise its just relationship with the "c"; the distance between them would be 680.448711 cents rather than a just perfect fifth (701.955001).

Example 12

In bar 10, the intonation of the root of chord V/II (D-Major) is calculated from the root of chord II (G-Major): 182.403712 (root of II) + 701.955001 (P5) = 884.358713 cents (root of V/II). In this example, a "g" root tuned to 203.910002 (a big M2 scale degree) would compromise its just relationship with the "d." The other notes are calculated as follows:

- f-sharp: 884.358713 (root of V/II) + 386.313714 (M3) = 1270.672427 ➔ 70.672427 cents ➔ ▽
- a: 884.358713 (root of V/II) + 701.955001 (P5) = 1586.313714 ➔ 386.313714 cents ➔ ∨

Rehearsal Phases to Develop Just Intonation

For each piece of music, just intonation is developed by progressing through four rehearsal phases: 1) Familiarization, 2) Realization, 3) Stabilization, and 4) Contextualization. Movement through these phases is fluid, but choral singing is a collective activity, and progression will only occur when most choristers are ready.

1. Familiarization Phase

For each piece of music, choristers can be introduced to the different intonations of each pitch class used. For example, in *Resonet in Laudibus,* two intonations of the notes g, b, d, and e-flat are required (as shown in the intonation notes below). If an annotated score is unavailable, choristers would be required to transfer symbols to their scores (they are not required to know cent values because these could be overwhelming). While there is personal time involved in this process, it is necessary to ensure that everyone is tuning in the same way.

Intonation Notes

Transfer the following just intonation tuning symbols to your *Resonet in Laudibus* scores:

- F is ○
- F-sharp is ▽
- G is ∨, except it is ↗ in the following instances:
  - Sopranos, bars 6, 11, 19, 22, 29 & 32
  - Altos, bars 13, 14, 23, 25 & 26
  - Tenors, bar 9
  - Basses, bars 11, 13, 14 & 26
- A is ∨
- B-flat is ↘
- B is ↘, except it is ▽ in the following instances:
  - Tenors, bars 8, 10, 15, 16, 17, 20, 27 & 30
- C is ↗
- D is ∨, except it is ↗ in the following instances:
  - Altos, bar 11
  - Tenors, bar 26
- E-flat is ↘, except it is ∧ in the following instance:
  - Tenors, bar 5 (2nd e-flat only)
- E is ∨

2. Realization Phase

Choristers then learn to tune their notes to the roots of chords by reading just intonation tuning symbols. They use their aural discernment skills and a physical sensation of resonance to confirm when their notes are in tune, specifically listening for no beats within and between classes of pitch. It is useful to settle the first chord of *Resonet in Laudibus* by using the tune-up process described below before asking choristers to sing it through slowly on a sustained /a/ vowel. This pedagogical approach is particularly useful for faster pieces and for music written in foreign languages; it gives singers time to focus on their tuning and avoids discrepancies in the intonation of different vowel resonances. S. Quinn wisely argues that "it is easier to tune a line through reiteration of one confirmed vowel sound than through the variety of changes which occur in the *solfa* syllables."[29]

During this phase, the conductor takes note of any intonation issues that arise, which can then be isolated and sorted during the next phase. To avoid the issue of diminishing returns in rehearsals, it is useful to divide longer works into shorter, more manageable units.

Tune-up Process

N. L. Norden provides some insight into how to approach the tuning of chords: "If there is any difficulty, first produce the octave in the chord….then add the fifth, and finally the third….This process should be used in fundamental position and all inversions."[30] Thus, the first chord of *Resonet in Laudibus* can be tuned using the following process:

1. Provide one starting pitch ("f" origin) vocally. Playing the starting note for each section on a piano is not recommended, as its temperament will corrupt the just intonation from the outset.

2. Basses tune their f3 root.

3. Altos match their f4 to the basses.

4. Tenors, and then sopranos, tune the perfect fifth (c4 and c5, respectively) to the intonation of the third harmonic produced by the basses (701.955001).

This method gives choristers time to think about the intonation required and respond accordingly; it also gives the conductor time to guide the singers through the process of aligning their intonation with the harmonics.

3. Stabilization Phase

The purpose of this phase is to stabilize any intonation "danger zones" observed by the conductor during the previous phase. The stabilization and realization phases share the same pedagogical approach: these zones are rehearsed slowly on a single vowel sound. It may be necessary to spend more than one rehearsal stabilizing intonation in some pieces of music, particularly if they are harmonically more complex.

4. Contextualization Phase

By this rehearsal phase, the just intonation should be well established in each singer's aural and muscle memories. During this final phase, choristers practice the multi-tasking that is required to sing in just intonation as they interpret other aspects of a score, such as tempo, dynamic, and articulation markings. Intonation is only rehearsed in isolation if the need arises.

Conclusion

Many writers on choral music education advocate tuning in just intonation.[31] The purpose of this article has been to provide an introduction for the use of just intonation in tuning unaccompanied choral music, particularly early music. There is historical evidence to suggest that just intonation was used prior to the evolution of Equal Temperament,[32] and in the Eighteenth-Century there was considerable debate about the negative effects that Equal Temperament had on music.[33] More importantly, however, scientific evidence shows that in comparison to temperaments, just intonation appears to offer an enhanced result in the tuning of unaccompanied tonal choral music. "[Chords] will 'stop' and shine with a special lustre, approx. like a slide picture that is correctly focused."[34] Unfortunately, what has been missing to this point is a tried and tested pedagogical approach for teaching choirs to sing in just intonation.

There are a number of factors that affect intonation, but the approach outlined in this article is a partial solution to the problem. The methodology has been tested with two choirs (and trialed with less-able choirs to good effect), and effectively taught the choristers to sing in just intonation while enhancing their ability to sing more reliably and consistently in tune. Specifically, all of the singers who participated in the post implementation survey reported positively about the effectiveness of the approach:

> I just wanted to thank you for your revolutionary work, and congratulate you on what has redefined music for me. Your work … has awakened my love of choral music and the beauty that can be attained. (Chorister A, personal communication, July 2016)

Choristers also mentioned some personal benefits of learning this system, such as increased confidence and musical knowledge: 92.5% reported that their intonation was enhanced, and 96.3% thought that their sense of intonation had improved. Their prompt tuning adjustments during rehearsals were a testimony to this.

Case study methods[35] were applied to this research to increase its reliability and validity, such as the collection and corroboration of data from various sources. Overall, the results showed that the approach effectively taught both choirs to sing in just intonation and enhanced the choristers' ability to sing more reliably and consistently in tune—90.2% of participants support this view. "[Just intonation tuning symbols] is an amazing system, and while I don't fully understand the calculations, the tuning is truly incredible," Chorister B, personal communication, July 2016.

Regarding potential bias: due to time and cost constraints, this research has been directed by the author with choirs conducted by the author. It could be argued that some of the participants' praise and gratitude was due to their personal support. However, subsequent to this research, this approach has also been beneficial for other choirs and their directors:

> [Just intonation tuning symbols] have been the most transformational tool I have experienced in my fourteen years of conducting school choirs. It has totally changed the way I listen to my choirs. It feels like another level of possibility has been unlocked for me… This transformation has not only happened for me but also my students. They have commented frequently that it feels "easier" to sing, and that they also listen completely differently. (Conductor A, personal communication, September 2018)

> I was initially concerned that just intonation would be both inaccessible and simply too difficult for our singers. However, using [just intonation tuning symbols], even with only a few of our pieces, greatly increased not just the precision of tuning, but generally the amount of listening our singers were doing (Conductor B, personal communication, July 2017)

Richard Strauss said: "The human voice is the most beautiful instrument of all, but it is the most difficult to play." Intonation is a significant contributor to this difficulty. All choirs will occasionally sing out of tune, but choral conductors may want to consider implementing just intonation as a tool toward a more consistent result. CJ

NOTES

1 Beats occur when there are different intonations within, or between, classes of pitch, that is, when different intonations of the same note occur simultaneously. For example, when an equally tempered perfect fifth (700 cents) and a just perfect fifth (701.955001 cents) are sounded together. N. L. Norden states that "beats are caused by interference between sound waves…" N. L. Norden, "A new theory of untempered music: A few important features with special reference to 'a cappella' music." *The Musical Quarterly* 22, no. 2 (1936): 217.

2 While harmonics are generated above every sung pitch, resultant tones are produced when the frequency of the notes (in Hertz) of harmonic intervals are added together (summation tones) or subtracted from each other (difference tones).

3 J. M. Barbour, "Just intonation." *Bulletin of the American Musicological Society* 2 (1937): 11.

4 J. Montegu, *Oxford Music Online.* "Just intonation," 2014. http://www.oxfordmusiconline.com/subscriber/article/opr/t114/e3611?q=Just+intonation&search=quick&pos=2&_start=1#firsthit.

5 N. L. Norden, "Untempered intonation." *Organ Insitute Quarterly* 3 (1953): 23.

6 P. Alldahl, *Choral intonation* (Stockholm: Gerhrmans Musikforlag, 2008): 33.

7 Based on the doctoral dissertation, Andrew Withington, "Enhancing choral intonation in unaccompanied tonal music: A curriculum and pedagogical approach to teach choirs to sing in just intonation." PhD diss., University of Canterbury, 2017.

8 D. M. Howard, "Intonation drift in a capella [sic] soprano, alto, tenor, bass quartet singing with key modulation." *Journal of Voice* 21, no. 3 (2007): 302.

9 K. D. Skelton, "Choral intonation." *Choral Journal* 46, no. 3 (2005): 43.

10 Howard, "Intonation drift," 303; and Skelton, "Choral intonation," 42.

[11] Norden, "Untempered intonation," 16.

[12] G. Averill, "Bell tones and ringing chords: Sense and sensation in Barbershop harmony." *The World of Music* 41, no. 1 (1999): 48.

[13] M. Sabat, "On Ben Johnston's notation and the performance practice of extended just intonation": 3. 2009. http://www.marcsabat.com/pdfs/EJItext.pdf.

[14] Norden, "Untempered intonation," 23.

[15] "Tuning notes of chords in a manner that produces a maximally consonant result is not trivial because it involves the use of relative pitches that are not found on today's keyboard instruments." D. M. Howard, H. Daffern, and J. Brereton, "Four-part choral synthesis system for investigating intonation in a cappella choral singing." *Logopedics, Phoniatrics, Vocology* 38, no. 3 (2013): 142.

[16] J. Marvin, "Choral singing, in tune." *Choral Journal* 32, no. 5 (1991): 27.

[17] "Whilst choral directors often focus on aspects of tuning when rehearsing ensembles, the lack of a thorough understanding of how tuning systems work can affect an overall tendency for the underlying pitch of the piece to change during the course of a piece of music (pitch drift) which can cause confusion in the development and aim of accurate tuning in choirs." Howard, Daffern, and Brereton (2013), 135.

[18] N. L. Norden, "A new theory of untempered music: A few important features with special reference to 'a cappella' music." *The Musical Quarterly* 22, no. 2 (1936): 220.

[19] B. Smith and R. T. Sataloff, *Choral Pedagogy* (San Diego: Plural Publishing, 2013): 274.

[20] J. Grant, "Improving pitch and intonation." *Choral Journal* 28, no. 5 (1987): 7.

[21] Other researchers have used symbols to show the just intonation of notes: B. Gratzki, *Die reine intonation im chorgesang* (Bonn: Verlag für systematische Musikwissenschaft GmbH, 1993): 245-267 uses (/) and (\) symbols; R. W. Duffin, "Cracking a centuries-old tradition" in *Early Music America* 20, no. 4 (2014): 46; and Duffin, "Just intonation in renaissance theory and practice" in *Music Theory Online* 12, no. 3 (2006): 11. uses positive (+1) and negative (–1) numbers, and Ben Johnston (according to Sabat, 2009, pp. 6-11) and Alldahl (2008, p. 10) use (+) and (–) signs. Alldahl also uses (○) for the key note.

[22] The words "sharp" and "flat" are not used to describe just intonation tuning symbols due to the negative connotations associated with these terms.

[23] C. Greated defines this as "the difference between a just major 3rd and four just perfect 5ths less two octaves [or a Pythagorean major 3rd], which is 21.51 cents." C. Greated, "Comma" in *The New Grove Dictionary of Music and Musicians,* edited by Stanley Sadie and John Tyrell. 2nd ed. (London: Macmillan, 2001): 170.

[24] For research referring to the "sensation" of singing in tune, see: Averill, "Bell tones and ringing chords," 49; Norden, "Untempered intonation," 19; Skelton, "Choral intonation," 43; Smith and Sataloff, *Choral Pedagogy,* 274; and T. Wine, "Check your intonation." *Choral Journal* 44, no. 9 (2004): 27.

[25] Alternatively, intonation notes can be prepared so choristers can mark their own scores.

[26] Roman numerals are used to indicate chords. First inversion chords are labelled using the letter "b" (Ib, viib, and so forth), and second inversion chords using the letter "c" (Ic, IVc, and so forth).

[27] J. M. Jordan and M. Mehaffey, *Choral Ensemble Intonation: Method, Procedures & Exercises* (Chicago: GIA, 2001): 15.

[28] Wine, "Check your intonation," 25.

[29] S. Quinn, "Choral intonation: A practical guide to the process and the development of skills necessary for acquiring and maintaining accurate tuning." *Canadian Music Educator* 37, no. 3 (1996): 9.

[30] Norden, "Untempered intonation." *Organ Insitute Quarterly* 3 (1953): 22.

[31] "We are still privileged—and obliged—in rehearsal to utilize nature's instincts and phenomena in order to 'tune up' our pitch relationships." H. Swan, "The development of a choral instrument" in *Choral Conducting, A Symposium,* 4-55 (New York: Appleton-Century-Crofts, 1973): 34.

[32] See: P. Barbieri and S. Mangsen, "Violin intonation: A historical survey." *Early Music* 19, no. 1 (1991): 69; and Alldahl, *Choral Intonation,* 33.

[33] S. M. Isacoff, *Temperament: The idea that solved music's greatest riddle* (Knopf, 2002): 4-6, 229.

[34] Alldahl, *Choral Intonation,* 17.

[35] Defined by R. Yin, *Case Study Research: Design and Methods.* 5th ed. (Los Angeles: SAGE, 2014).


Choir 1
Choir 2
Both Choirs
Soprano
Alto
Tenor
Bass
Root Line
F ○
Re - so-net in lau - di-bus, cum ju-cun - dis plau - si-bus, Si - on cum fi - de - li-bus: Ap-
pa - ru - it quem ge - nu - it Ma - ri - - a. Sunt im-ple - ta quae prae-di - xit
Ga - bri - el. E - ia! E - ia! Vir - go De - um ge - nu - it,
Ga - bri - el. E - ia, e - ia! E - ia, e - ia! Vir - go De - um ge - nu - it,
Choir 1
S
A
T
B
7
12


Choir 2
17
quod di - vi - na vo - lu - it cle - men - ti - a, quod di - vi - na vo - lu - it cle - men - ti - a.
Both Choirs
23
Ho - di - e ap - pa - ru - it, ap - pa - ru - it in Is - ra - el, ex Ma - ri - a
28
Vir - gi - ne est na - tus Rex, ex Ma - ri - a Vir - gi - ne est na - tus Rex.
S
A
T
B